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Research Article

Approximate Effective Interaction for Nuclear Matter and Finite Nuclei

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Abstract

In this paper, an approximate effective nucleon-nucleon interaction for nuclear matter and finite studies has been derived using the lowest order constrained variational (LOCV) approach. The LOCV method, a functional minimization procedure, uses a normalization constraint to keep higher-order terms as small as possible. As a first step, two-body matrix elements based on the Reid93 nucleon-nucleon potential were calculated for the nuclear system A = 16 in a harmonic oscillator basis, with the oscillator size parameter $\hbar\omega$ = 14.0 MeV, and separated into the central, spin-orbit and tensor channels in conformity with the potentials for Inelastic scattering. Following this, a least squares fitting of the matrix elements to a sum of Yukawa functions was performed to determine the strengths of the effective interaction in the singlet-even, singlet-odd, triplet-even and triplet-odd (Central); tensor-even and tensor-odd (Tensor); spin-orbit-even and spin-orbit-odd (Spin-orbit) channels. Of all the matrix elements, only the triplet-even and tensor-even components, being attractive, are affected by the tensor correlations (a = 0.05); and are shown to exhibit the same trend of variation in conformity with past work, in terms of magnitude, as one goes from the lower-node quantum numbers (n', n) = (0, 0) to higher ones (n', n) = (2, 2). When compared with the G-matrix results of previous researchers, the results obtained herein have been found to be in good agreement. This, therefore, gives hope that the new effective interaction promises to be a reliable tool for nuclear matter and nuclear structure studies.

Keywords

Matrix Elements, Effective Interaction, Nuclear Matter, Finite Nuclei

1. Introduction

Effective nucleon-nucleon (NN) interactions have been at the centre of Nuclear Physics researches since the discovery of the neutron because of their crucial role in nuclear matter calculations. In recent years, a number of effective interactions have been successfully developed based on approaches ranging from empirical fit of experimental data to microscopic derivation from the bare NN potential. Some of these approaches are the G-matrix approach [1], lowest-order constrained variational (LOCV) principle [2], relativistic mean-field theory (RFM) [3], Chiral quark models based on quantum chromodynamics (QCD) [4] and effective field theory (EFT) [5].

A very useful feature of these interactions is that analytical expressions for many interesting quantities in both symmetric and asymmetric nuclear matter are contained in them, making the study of nuclear matter properties a transparent exercise.

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Generally, the effective NN interactions can be divided into two groups [6]. The first group is that in which the effective NN interaction is directly parametrized as a whole, leaving out any connection with a realistic free NN interaction. In the non-relativistic approaches, the parameters of the effective NN interaction belonging to this group are obtained by fitting the Hartree-Fock (HF) mean-field result to experimental data. The second group involves the derivation of the effective NN interaction in the lowest order of many-body calculation from a realistic free NN scattering data (e.g a solution of Be-the-Goldstone equation) and the higher-order corrections are parametrized in terms of a density or momentum dependence [7-9]. One of the effective interactions belonging to the first group is the Skyrme interaction [6] while the M3Y-Reid and M3Y-Paris effective interactions are examples of effective interactions belonging to the second group. The effective interactions [2], developed based on the Reid68 [10] nucle-on-nucleon (NN) potential within the framework of cluster expansion technique via the LOCV approach by using two-body correlation functions, belong to the second group. These inter-actions [2], have continued to be applied to nu-clear matter [11-13] and nuclear reaction [14-17] studies with excellent results. Our findings from these studies have shown that very simple two-body correlation functions can be used to obtain very accurate results comparable with the best availa-ble methods.

The ultimate goal of this work is to update our previous calculations [2] based on the fact that improvement in NN phase-shift data has led to the construction of more accurate and high quality NN potentials, one of which is the Nijmegen potential known as the Reid93 [18, 19]. This work is also an update on the earlier calculations [20]. Doing this, the nuclear matrix elements of the two-body interaction, leading to our approximate effective interaction, are first derived within the framework of LOCV using two-body correlation functions based on the Reid93. When successfully developed, the ap-proximate effective interaction will be applied to nuclear matter and nuclear reactions in subsequent calculations with the intent to compare its performance with that of the effective interactions [2]; and this should show clearly the effect of the improved phase shift on the new effective interaction. Knowing fully well that the LOCV method parallels the G-matrix approach or any other sophisticated approach to the microscopic study of effective interactions, the performance of the new interaction will be subsequently compared with those of the G-matrix effective interactions and the like.

2. The Lowest Order Constrained Variational Approach

The lowest order constrained variational (LOCV) method employs a normalization constraint to keep higher-order terms as small as possible [21]. This functional minimization procedure represents an enormous computational simplifica-tion over unconstrained methods that attempt to go beyond lowest order.

To obtain an effective two-body interaction, the LOCV method was employed by Fiase and his co-researchers [2] who have shown that a non-relativistic nucleon fluid inter-acting through a two-body potential, in the centre of mass rest-frame, can be represented by the Hamiltonian:

$$H_{ij} = \sum_i -\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i>j} V_{ij} \tag{1}$$

where V_{ij} is the two-body potential and the translationally invariant component of the trial wave function is defined as:

$$\psi_T = UG\phi \tag{2}$$

with U as a unitary operator which transforms the system to the centre-of-mass rest frame, leaving only the intrinsic quantities so that spurious centre-of-mass motion is no longer a matter of concern. G is a symmetric product of two-body correlation functions defined as [22]:

$$G = \prod_{i>j} g_2\,(ij) \tag{3}$$

where $g_2(ij)$ are the two-body correlation operators. These correlations are formulated to accommodate the effect of the strong repulsion of the nucleon-nucleon interaction, and ϕ is a multidimensional product of two-body wave functions. The explicit form of $g_2(ij)$, the two-body correlation operators is given in equations (8-11) below.

The multidimensional nature of the Hamiltonian in equa-tion (1) makes it difficult to calculate its matrix elements, so approximations must be made using the cluster expansion technique in which the system is divided into clusters, re-sulting in a two-body effective interaction. This way, the energy of each cluster is evaluated starting with the two-body clusters and then summed over all the clusters to obtain the total energy:

$$E = E_2 + E_3 + E_4 - - - \tag{4}$$

In this approximation, only the two-body clusters are most important, so the two- body energy term E2 is minimized to the lowest order with respect to the functional variations of the two-body correlation functions while the contributions of E3 and higher order clusters are made negligibly small. This approximation implies that one is concerned only with the two-body energy term, E_2 of the form [23]:

$$E_2 = \langle\phi\left|\sum_{i>j}\left[g_2(ij)\left(\frac{P_{ij}^2}{M} - V_{ij}\right)g_2(ij)\right]\right|\phi\rangle, \tag{5}$$

where $P_{ij} = \frac{1}{\sqrt{2}}\left(P_i - P_j\right)$ is the relative momentum of the two-particle system.

$M \sim m\ NA$ is the total mass of the nucleus, and V_{ij} is the

Reid93 [18, 19] potential of the form:

$$V_{ij} = \sum V_{ij}^{\theta} \tag{6}$$

where θ are different reaction channels with the central, spin-orbit and tensor components. For each channel, the two-body correlation function has a form consistent with that of the chosen potential [24]. Therefore,

$$g_2(ij) = \sum_{\theta} g_{ij}^{\theta} \tag{7}$$

where,

$$g_{ij} = g_C^{\theta}(x_{ij}) + g_{LS}^{\theta}(x_{ij}) L \cdot S + g_T^{\theta}(x_{ij}) S_{ij} \tag{8}$$

and the tensor operator [25]:

$$S_{ij} = 3(\sigma_i . x_{ij})(\sigma_j . x_{ij}) - \sigma_i \sigma_j \tag{9}$$

Studies on nuclear matter and finite nuclei [22] have revealed the two-body correlation functions to have the tensor correlations in the $^3S_1$ - $^3D_1$ channel as the most important feature. Accordingly, the two-body correlation functions of equation (8) have been parameterized in the form [26]:

$$g_2(ij) = 0, x_{ij} < x_C \tag{10}$$

$$g_2(ij) = \left(1 - e^{-\beta(x_{ij} - x_C)^2}\right)\left[1 + \alpha^{\lambda}(A) S_{ij}\right], x_{ij} > x_C$$

where x_C = 0.25fm and β =25 fm - 2. The parameter $\alpha^{\lambda}(A)$ represents the strength of the tensor correlations and is non-zero only in the lowest coupled $^3S_1$ - $^3D_1$ channel.

To evaluate matrix elements, one works in the harmonic oscillator basis. In this regard, only the effective two-body potential energy terms are obtained from equation (6) as:

$$E_2' = \langle \phi | \sum_{i>j} [g_2(ij) V_{ij} g_2(ij)] | \phi \rangle \tag{11}$$

with the oscillator size parameter $\hbar\omega$ and the strength of the tensor correlations $\alpha^{\lambda}(A)$ as the only parameters appearing in the calculation.

Now, following the procedure of [1], the two-body potential matrix elements are separated into their respective channels. These are the singlet-even (SE) and singlet-odd (SO) channels denoted by $^1S_0$ and $^1P_1$ respectively. The triplet even (TE) and tensor-even (TNE) components are picked from the coupled $^3S_1$ - $^3D_1$ channel. The triplet-odd (TO), tensor-odd (TNO) and the two components of the spin-orbit force are defined as [27]:

$$V(TO) = V(3P_0) + 2V(LSO) + 4V(TNO) \tag{12}$$

$$V(TNO) = -\frac{5}{72}[2V(3P_0) - 3V(3P_1) + V(3P_2)]$$

$$V(LSO) = -\frac{1}{12}[2V(3P_0) + 3V(3P_1) - 5V(3P_2)]$$

$$V(LSE) = \frac{1}{3}[V(TE) - 2V(TNE) - V(3D_1)]$$

3. The Approximate Effective NN Interaction

This Section is devoted to the definition of the effective NN interaction for nuclear matter calculation and finite nuclei studies.

The effective nucleon-nucleon interaction has three potential components which are the central (V_C), spin-orbit (V_{LS}) and tensor (V_T) components [1, 2]. These are expressed as follows:

$$V_C = \sum_k V_k Y\left(\frac{x_{ij}}{R_k}\right) \tag{13}$$

$$V_{LS} = \sum_k V_k Y\left(\frac{x_{ij}}{R_k}\right) L \cdot S$$

$$V_T = \sum_k V_k x_{ij}^2 Y\left(\frac{x_{ij}}{R_k}\right) S_{ij}$$

where $Y\left(\frac{x_{ij}}{R_k}\right)$ is a Yukawa potential function of the form:

$$Y\left(\frac{x_{ij}}{R_k}\right) = \frac{exp\left(-\frac{x_{ij}}{R_k}\right)}{\left(\frac{x_{ij}}{R_k}\right)} \tag{14}$$

V_k in equations (13) are the strengths of the effective interaction to be determined by fitting the two-body matrix elements of Equation (11) to those of a sum of Yukawa functions calculated in a harmonic oscillator basis with different ranges; R_k are the ranges which are chosen to be 0.25, 0.40, 0.70 and 1.414 fm [1, 27]; and x_{ij} is the separation between the i and j nucleons.

The central potential, V_C is purely radial, depending only on the coordinate x of the two nucleons [28, 29]. For this component, the ranges used are R_1= 0.25, R_2 = 0.40 and R_3 = 1.414 fm [1, 27]. The third and longest range of the potential corresponds to the exchange of a π-meson, which represents the one-pion exchange potential (OPEP), the second is meant for the simulation of multiple pion exchange and the third is chosen to improve the fit [27].

The tensor potential, V_T is non-central and depends on the relative position vector, x and the relative spin orientations, S_1 and S_2. The tensor operator S_{12} in the potential has the form shown in Equation (9). The ranges for the computation of this component are R_1 = 0.40 and R_2 = 0.7 fm.

The spin-orbit potential is consequent upon the interaction between the orbital angular momentum L and spin angular momentum S represented by L·S. The potential arises from the coupling of L and S to give the total angular momentum J

given by the equation: $J^2 = (L+S)^2 = L^2 + S^2 + 2L \cdot S$. The spin-orbit force has an expectation value proportional to [28, 29]: $2\langle L \cdot S \rangle = j(j+1) - l(l+1) - s(s+1)$, where $L = \sqrt{l(l+1)}$, $S = \sqrt{s(s+1)}$, $J = \sqrt{j(j+1)}$. Since the spin-orbit force has a short range, the ranges chosen for the spin-orbit potential are R i = 0.25 and R 2 = 0.40 fm [1, 27].

As presented in Section 2, the effective interaction derived herein is based on the LOCV method with the two-body matrix elements of Equation (11) fitted to a sum of Yukawa functions, resulting in interaction strengths, which are separated into various angular momentum channels; namely, the singlet-even (SE), singlet-odd (SO), triplet-even (TE), triplet-odd (TO) along with spin-orbit and tensor channels. This is shown in the results presented in Section 4. This procedure has been used to construct the popular G-matrix-based M3Y-Reid [1] and M3Y-Paris [27] as well as the B3Y-Fetal [2] derived from LOCV approach. These effective interactions have been successfully used in various nuclear matter [11, 12, 17] and nuclear reaction [13-16] studies. Similarly, the new interaction will hopefully, subsequently be put to use in nuclear matter and nuclear reactions so as to determine its performance strength in comparison with the other effective interactions.

4. Results and Discussion

In this work, an approximate effective interaction for nuclear matter and finite nuclei studies has been derived from the basic nucleon-nucleon (NN) force via the intermediary of a representative set of matrix elements based on the Reid93 potential using the lowest order constrained variational approach (LOCV). The matrix elements were computed for the nuclear system A = 16 in a harmonic oscillator basis, with $\hbar\omega$ = 14.0 MeV, and fitted to a sum of Yukawa terms to determine the strengths of the effective interaction in the various angular momentum channels. In all, the results presented here involve eight angular momentum channels, which are SE, SO, TE and TO (central); TNE and TNO (tensor); and LSE and LSO (spin-orbit) channels. The relative matrix elements are presented in Table 1, whereas their accompanying strengths of interaction are displayed in Table 2. Generally, the results of the present calculation are shown to demonstrate good agreement with the G-matrix results [1, 27].

Table 1. *Calculated Matrix Elements for A = 16, ħω = 14.0 MeV and α = 0.05.*

SE				**TE**			
(S/S)	**n = 0**	**1**	**2**	**(S/S)**	**n = 0**	**1**	**2**
n' = 0	-7.69	-6.75	-5.49	n' = 0	-10.50	-8.59	-6.49
	(-6.64)	(-5.41)	(-3.80)		(-9.93)	(-8.84)	(-6.81)
1		-6.63	-5.60	1		-8.32	-6.76
		(-4.73)	(-3.30)			(-7.84)	-5.71)
2			-4.96	2			-6.03
			(-2.23)				(-3.76)
SO				TO			
(P/P)	n = 0	1	2	(P/P)	n = 0	1	2
n' = 0	2.54	2.66	2.61	n' = 0	0.52	0.62	0.72
	(2.52)	(2.37)	(2.11)		(-0.08)	(-0.23)	(-0.09)
1		3.56	3.74	1		0.94	1.14
		(3.03)	(3.00)			(-0.24)	(-0.05)
2			4.30	2			1.45
			(-2.23)				(0.04)
TNE				TNO			
(S/D)	n = 0	1	2	(P/P)	n = 0	1	2
n' = 0	-5.07	-7.06	-8.22	n' = 0	0.71	0.70	0.61
	(-5.76)	-8.12)	(-9.71)		(0.78)	(0.75)	(0.63)

SE				TE			
(S/S)	**n = 0**	**1**	**2**	**(S/S)**	**n = 0**	**1**	**2**
1	-2.50	-4.84	-6.61	1		0.85	0.81
	(-2.81)	(-5.52)			(0.92)	(0.87)	
2	-1.28	-2.77	-4.45	2			0.87
	(-1.52)	(-3.23)	(-5.30)				(0.95)
LSE				LSO			
(D/D)	n = 0	1	2	(P/P)	n = 0	1	2
n' = 0	-0.26	0.20	0.92	n' = 0	-0.43	-0.74	-0.98
	(-0.14)	(-0.13)	(-0.12)		(-0.60)	(-0.89)	(-1.06)
1		0.30	0.74	1		-1.13	-1.45
		(-0.18)	(-0.21)			(-1.26)	(-1.52)
2			-0.96	2			-1.81
			(-0.26)				(-1.84)

The results in parentheses for SE, TE, TNE and LSO channels are the G-matrix results [1] while those in parentheses for the SO, TO, TNO and the LSE channels are the G-matrix results [27].

Table 2. *Best-Fit Interaction Strengths (MeV) for A =16, ħω = 14.0 MeV and a = 0.05. The results in parentheses for SE, TE, TNE and LSO channels are the G-matrix results [1] while those in parentheses for the SO, TO, TNO and the LSE channels are the G-matrix results [27].*

S/N	Channel	R_1 = 0.25 fm	R_2 = 0.40 fm	R_3 = 0.70 fm	R_4 = 1.414 fm
1	SE	15228.20	-5025.36		-10.463
		(12455)	-3835		(-10.463)
2	TE	18258.78	-5800.38		-10.463
		(21227)	(-6622)		(-10.463)
3	SO	-17.92	1465.22		31.389
		(-1418)	(950)		(31.389)
4	TO	10066.15	-1048.60		3.488
		(11345)	(-1900)		(3.488)
5	TNE		-1062.2	-26.71	
			(-1369)	(-10.69)	
6	TNO		232.79	13.73	
			(-19.71)	(27.06)	
7	LSE	-111.27	3085.20		
		(-5101)	(-337)		
8	LSO	-5911.92	93.19		
		(-2918)	(-483)		

In the central channels, the computed two-body matrix elements for SE, TE and SO are shown in Table 1 to be in a better agreement with the G-matrix than the TO matrix elements which are much larger than their G-matrix counterparts. The central-even components, being attractive and in good agreement with the results [27], are seen to exhibit the same trend of variation, in terms of magnitude, as one goes from the lower-node quantum numbers (n', n) = (0, 0) to higher ones (n', n) = (2, 2), but only the TE channel is affected by the tensor correlations *(a* = 0.05) believed to be responsible for the negative sign of the matrix elements. Concerning the central-odd components, the big difference between the TO matrix elements and their G-matrix counterparts is traceable to the different computational approach used for their evaluation in the present work.

As regards the tensor channels, the TNE matrix elements are affected by the tensor correlations; and they are found to be attractive and stronger than their repulsive TNO counterparts in acceptable agreement with the G-matrix results [1, 27]. It is noteworthy that the observed agreement is indicative of the reliability of the method used herein.

Pertaining to the spin-orbit channels, it is evident from Table 1 that the LSO channel is in excellent agreement with the results of Anantaraman and co-researchers [27] as both matrix elements are strikingly similar in sign and magnitude. But, the LSE channel, being repulsive in the present calculations, is noticeably different from [27], with matrix elements that are considerably larger than the latter.

The interaction strengths presented in Table 2 have resulted from a least squares fitting of the calculated two-body matrix elements to the oscillator matrix elements of a Yukawa sum in the central, tensor and spin-orbit channels respectively. The ranges of 0.25, 0.40, 0.70 and 1.414 fm, theoretically motivated, were chosen in conformity with the potential for inelastic scattering [1], with the one-pion exchange potential (OPEP) strength imposed on the 1.414 fm part of the central interaction as prescribed by the meson theory (exchange model) of nuclear forces. As shown in Table 2, the results of the present work are in impressive agreement with those of the G-matrix [1, 27] in most of the reaction channels. This gives hope that the new effective interaction promises to be a reliable tool for nuclear matter and nuclear structure studies. Efforts will be made to verify this quantitatively in the subsequent papers.

5. Conclusion

Based on the LOCV approach, with the Reid93 nucleon-nucleon potential folded with two-body correlation functions, an approximate effective interaction has been derived herein. Comparing the effective interaction with its G-matrix counterparts in the various angular momentum channels, it has been found to demonstrate excellent agreement with them. This success paves the way for us to apply the new effective interaction to nuclear structure studies in subsequent papers.

Abbreviations

NN	Nucleon Nucleon
LOCV	Lowest Order Constrained Variational
RFM	Relativistic Mean Field
EFT	Effective Field Theory
QCD	Quantum Chromodynamics
HF	Hartree-Fock
M3Y	Michigan-3-Yukawa
SE	Singlet Even
SO	Singlet Odd
TE	Triplet Even
TNE	Tensor Even
TNO	Tensor Odd
LSE	Spin-Orbit Even
LSO	Spin-Orbit Odd
OPEP	One-Pion Exchange Potential

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Conflicts of Interest

The authors declare no conflicts of interest.

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