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Practical Insight of Ferroconvection in Heterogeneous Brinkman Porous Medium

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Abstract: Ferromagnetic fluids are made up of magnetic particles, which are suspended in a carrier liquid such as water, hydrocarbon (mineral oil or kerosene) or fluorocarbon with a surfactant to avoid clumping. Worldwide many literature revealed that, the ferromagnetic fluids application has been diversified in nature and widely used in engineering, technology, agricultural, animal and biomedical sciences etc. (evidence based medicine for cancer patients, fertigation in agriculture). Now a days, the driven applications are using in developing countries. The ferromagnetic fluids analytical applications are very limited scope in Indian scenario due to paucity of literature and technological gap. In the essence of this research gap the present study undertaking to demonstrate the various applications of ferroconvection in a heterogeneous Brinkman porous medium on theoretical basis. The resulting eigenvalue problem is solved numerically using the Galerkin method. The effects of vertical heterogeneity of permeability, Darcy parameter, Magnetic Rayleigh number, nonlinearity of magnetization, and internal heat source on the onset of ferromagnetic convection is investigated.

Keywords: Heterogeneous Porous Medium, Ferrofluid, Ferromagnetic Convection, Variable Permeability, Internal Heat Source

1. Introduction

The ferromagnetic fluids are made up of magnetic nanoparticles, which are suspended in a carrier liquid such as water, hydrocarbon (mineral oil or kerosene) or fluorocarbon with a surfactant to avoid clumping [1, 2]. Ferrofluids possess with an extensive applications in several fields ranging from physics, electronics, electrical engineering, bio-medical, micro and nanoelectromechanical systems, instrumentation in computer technology and various heterogeneous engineering applications [3-5]. The magnetization of ferrofluids will depend on the magnetic field, temperature and density. Since, the magnetic forces have propagated thermal state of fluid and it was derived from the ferromagnetic convection. Hence, the heat can transfer by ferromagnetic fluids and it will be emerged as one of the major areas to know the various applications of engineering sciences. The problem of ferromagnetic convective instability leads to magnetized ferrofluid layer, it was heated from below minimal temperature, this was investigated by Neuringer et al. Rosensweig et al. Finlayson et al. Recently, Afifah et al. studied various applications of ferroconvection in a magnetized ferrofluid with saturating porous medium. In Indian context a similar study was reported by Sadrhosseini et al he observed that, the heat can transfer to ferrofluid by porous media inside canal with uniform heat flux on the wall and its effect was seen in magnetic field [6-9].

However, the ferromagnetic fluids analytical applications is very limited scope in worldwide due to paucity of literature and technological gap, there is a scope for further investigations on ferromagnetic convection in a heterogeneous Brinkman porous medium with internal heat source [10-12]. In this wide research gap, the present study is attempt to demonstrate the various applications of ferromagnetic convection with onset of penetrative ferromagnetic convection in a ferrofluid-saturated horizontal heterogeneous Brinkman porous layer (uniformly distributed internal heat source).

2. Mathematical Formulation

Consider an incompressible magnetized ferrofluid-saturated infinite horizontal Brinkman heterogeneous porous layer of thickness d with the presence of a uniform applied magnetic field $(0,\ 0,\ H_0)$ in the vertical direction. The lower surface was held at constant temperature T_L, while the upper surface is at T_U $(<T_L)$. A Cartesian co-ordinate systems (*x, y, z*) used with the origin at the bottom of the porous layer and the z-axis directed vertically upward in the presence of gravitational field $\vec{g}$. In addition to that, the model is uniformly distributed with internal heat source in the ferrofluid saturated heterogeneous porous layer. The Boussinesq approximation density was estimated from the following equation.

$$\nabla\cdot\vec{q}=0 \quad \rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q}\right]=-\nabla \mathrm{p}+\rho_f\vec{g}-\frac{\mu_f}{K(z)}\vec{q}+\tilde{\mu}_f\ \nabla^2\vec{q}+\mu_0(\vec{M}\cdot\nabla)\vec{H} \quad A\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa\nabla^2T+Q \quad (1)$$

$$\nabla.\vec{B}=0$$

$$\nabla\times\vec{H}=0 \text{ or } \vec{H}=\nabla\varphi$$

$$\vec{B}=\mu_0(\vec{M}+\vec{H})$$

$$\vec{M}=M(H,T)\frac{\vec{H}}{H}$$

$$M=M_0+\chi(H-H_0)-K_p(T-T_a) \quad (2)$$

where, $\vec{q}$ denotes velocity vector, p - pressure, ρ_f - fluid density, $K(z)$ -variable permeability of the porous medium, $\vec{M}$ -magnetization of the ferrofluid, $\vec{B}$ - magnetic induction, $\vec{H}$ - magnetic field, μ_f - fluid viscosity, $\tilde{\mu}_f$ - effective viscosity, μ_0 - magnetic permeability, ρ_0 - reference density, T - temperature, A - ratio of heat capacities, ε - porosity of the porous medium, κ - thermal diffusivity, Q- overall uniformly distributed effective volumetric internal heat generation, α_t -thermal expansion coefficient, $\chi=(\partial M/\partial H)_{H_0,T_a}$ -expressed magnetic susceptibility, $K_p=-(\partial M/\partial T_f)_{H_0,T_a}$ derived the pyromagnetic co-efficient,

$$M_0=M(H_0,T_a),\ T_a=(T_L+T_U)/2,\ H=\left|\vec{H}\right|,\ M=\left|\vec{M}\right|\ \varphi \quad (3)$$

$\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$ real values of Laplacian operator

In the basic conduction state, the following equation was formulated

$$\vec{q}_b=0$$

$$p_b(z)=p_0-\rho_0 g\,z-\rho_0\alpha_t g\left[\frac{Qz^3}{6k}-\frac{Qdz^2}{4k}+\beta\left(\frac{z^2}{d}-\frac{zd}{2}\right)\right]-\frac{\mu_0M_0K_p}{1+\chi}\left[\frac{Qz^2}{2k}-\frac{Qdz}{2k}+\beta z\right]$$
$$-\frac{\mu_0K_p^2\beta^2}{(1+\chi)^2}\left[\frac{Q^2z^4}{8k^2}+\frac{Qz^3}{4k}\left\{2\beta-\frac{Qd}{k}-\frac{\beta d}{z}\right\}+\frac{Qdz^2}{8k}\left\{\frac{Qd}{k}-4\beta+\frac{2\beta d}{z}\right\}+\frac{\beta^2}{2}\left(z^2-z^2d\right)\right]$$

$$T_b(z)=T_a-\beta\left(z-\frac{d}{2}\right)-\frac{Q}{2\kappa}z^2+\frac{Qd}{2\kappa}z$$

$$\vec{H}_b(z)=\left[H_0-\frac{K_p}{1+\chi}\left\{\frac{Q}{2\kappa}z^2-\frac{Qd}{2\kappa}z+\beta\left(z-\frac{d}{2}\right)\right\}\right]\hat{k}$$

$$\vec{M}_b(z) = \left[M_0 + \frac{K_p}{1+\chi} \left\{ \frac{Q}{2\kappa} z^2 - \frac{Qd}{2\kappa} z + \beta \left(z - \frac{d}{2} \right) \right\} \right] \hat{k} \tag{4}$$

where, $\beta = \Delta T / d = (T_L - T_U) / d$ is the temperature gradient, $\hat{k}$ - unit vector in the z-direction and the subscript b clearly expressed the basic state. Later the equation was superimposed with perturbations, the basic solution becomes

$$\vec{q} = \vec{q}',\ p = p_b + p',\ T = T_b + T',\ \vec{H} = \vec{H}_b + \vec{H}' \text{ and } \vec{M} = \vec{M}_b + \vec{M}' \tag{5}$$

The linear stability analysis was performed with normal mode; non-dimensionalising of the real variables tends to be in the following form of equation

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d} \right),\ t^* = \frac{\kappa}{d^2} t,\ W^* = \frac{d}{\kappa} W,\ \Theta^* = \frac{1}{\beta d}\ \Theta,\ \Phi^* = \frac{(1+\chi)}{K_p \beta d^2} \Phi \tag{6}$$

Non-dimensional governing equations (asterisks for simplicity and noting that the principle of exchange of stability holds) are derived in the form of

$$\left[Da\left(D^2 - a^2 \right) - F(z) \right]\left(D^2 - a^2 \right) W - DF(z) DW$$
$$= R_m a^2 \left[N_s (1 - 2z) - 1 \right] (D\Phi - \Theta) + R_D a^2 \Theta$$

$$\left(D^2 - a^2 \right) \Theta = [N_s (1 - 2z) - 1] W \quad \left(D^2 - a^2 M_3 \right) \Phi - D\Theta = 0\,. \tag{7}$$

In the above equations, $D = d / dz$ is the differential operator, a can explain horizontal wave number $R_D = \alpha_t g \beta K_0 d^2 / \nu \kappa$ and Darcy-Rayleigh number was given by the equation $Da = \tilde{\mu}_f K_0 / \mu_f d^2$ and also the modified Darcy number was propagated in the form of $M_1 = \mu_0 K_p^2 \beta / (1+\chi) \alpha_t \rho_0 g$ $R_m = R_D M_1 = \mu_0 K_p^2 \beta^2 d^2 / (1+\chi) \mu \kappa$. Finally after algebraic solution, we have obtained the Darcy-Rayleigh number equation $N_s = Qd / 2\kappa\beta$; where $M_3 = (1 + M_0 / H_0) / (1+\chi)$ is the measure of non-linearity of magnetization and $F(z) = K_0 / K(z)$ non-dimensional permeability heterogeneity function and K_0 is the mean value of $K(z)$. The function $F(z)$ was chosen in the following form of equation

$$F(z) = 1 + \alpha_1 \left(z - \frac{1}{2} \right) + \alpha_2 \left(z^2 - \frac{1}{3} \right) \tag{8}$$

where α_1 and α_2 is real valued constants and it was formulated in quadratic function with unit mean. For the homogeneous porous medium case $\alpha_1 = 0 = \alpha_2$.

Equations (7) - (8) we have solved that, the appropriate boundary conditions tend to be significantly homogeneous. The simulated boundaries are found to be rigid, ferromagnetic state, either in isothermal or insulated temperature perturbations. The boundary conditions was estimated in the following equation

$$W = DW = \Theta \text{ or } D\Theta = \Phi = 0 \text{ at } z = 0, 1\,. \tag{9}$$

The resulting eigenvalue problem was solved numerically by using Galerkin method. Accordingly, $W(z)$, $\Theta(z)$ and $\Phi(z)$ would be expanded in the series form

$$W = \sum_{i=1}^{n} A_i\, W_i(z)\,,\ \Theta(z) = \sum_{i=1}^{n} B_i\ \Theta_i(z)\,,\ \Phi(z) = \sum_{i=1}^{n} C_i\ \Phi_i(z) \tag{10}$$

where A_i, B_i and C_i are unknown coefficients. Multiplying Equation by $W_j(z)$, $\Theta_j(z)$ and $\Phi_j(z)$ respectively and integrate them across the layer. Using the boundary conditions, we obtain the following system of linear homogeneous algebraic equations

$$C_{ji} A_i + D_{ji} B_i + E_{ji} C_i = 0$$

$$F_{ji} A_i + G_{ji} B_i = 0$$

$$H_{ji} B_i + I_{ji} C_i = 0\,.$$

The coefficients C_{ji} - I_{ji} can involve inner products of the basic functions and are given by

$$C_{ji} = Da\left[< D^2W_jD^2W_i > +2a^2 < DW_jDW_i > +a^4 < W_jW_i >\right]$$
$$+ <\left(1+\alpha_1(z-1/2)+\alpha_2(z^2-1/3)\right)DW_j\,DW_i > - <\left(\alpha_1+2\alpha_2 z\right)W_j\,DW_i >$$
$$+a^2 <\left(1+\alpha_1(z-1/2)+\alpha_2(z^2-1/3)\right)W_j\,W_i >$$
$$D_{ji} = -R_D\,a^2 < W_j\,\Theta_i > + R_m a^2 < \left[N_s\,(1-2\,z)-1\right]W_j\,\Theta_i >$$
$$E_{ji} = -R_m a^2 < \left[N_s\,(1-2\,z)-1\right]W_j D\Phi_i >$$
$$F_{ji} = < \left[N_s\,(1-2\,z)-1\right]\Theta_j W_i >,\ G_{ji} = < D\Theta_j D\Theta_i > + a^2 < \Theta_j\Theta_i >$$
$$H_{ji} = -< D\Phi_j\Theta_i >,\ I_{ji} = < D\Phi_j D\Phi_i > + a^2 M_3 < \Phi_j\Phi_i > \tag{11}$$

Where, the inner product is defined as $< \cdots > = \int_0^1 (\cdots)\,dz.$

The base functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ is assumed in the following form:

$$W_i = (z^4 - 2\,z^3 + z^2)T_{i-1}^*,\ \Phi_i = (z^3 - 3z^2 + 2z)T_{i-1}^*$$
$$\Theta_i = (z^2 - z)T_{i-1}^*\ \text{(isothermal)},\ \Theta_i = z^2(2z-3)T_{i-1}^*\ \text{(insulated)}, \tag{12}$$

where, T_i^* s ($i \in N$) is the modified Chebyshev polynomials, $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ will satisfies the corresponding boundary conditions. The characteristic equation was formed from (11) - (12) with the existence of non-trivial solution.(solved numerically with different values of M_3, N_s, R_m, Da and for different forms of $F(z)$). It was observed that, the numerical results were converged by taking sixth terms of Galerkin expansion. In the interest of comparison equations (8) – (9) were pooled with boundary conditions and real intervention was solved numerically by using shooting technique (Runge-Kutta-Fehlberg and Newton-Raphson iteration methods). The equation (8) – (9) was solved by considering initial value of $z = 0$

For isothermal boundaries

$$W(0)=0,\ DW(0)=0,\ D^2W(0)=1,\ D^3W(0)=\eta_1$$
$$\Theta(0)=0,\ D\Theta(0)=\eta_2$$
$$\Phi(0)=0,\ D\Phi(0)=\eta_3$$

For insulated boundaries

$$W(0)=0,\ DW(0)=0,\ D^2W(0)=\eta_1,\ D^3W(0)=\eta_2$$
$$\Theta(0)=1,\ D\Theta(0)=0$$
$$\Phi(0)=0,\ D\Phi(0)=\eta_3.$$

Here, the conditions $D^2W(0)=1$ and $\Theta(0)=1$ help us to break the scale invariance of the solution of isothermal and adiabatic boundaries respectively. Further, the parameters of η_1, η_2 and η_3 are unknown, parameters were determined from the Darcy Rayleigh number R_D at isothermal and insulated temperature boundary condition. The shooting method was obtained by Runge-Kutta-Fehlberg method (RKF45), the formulation of the fitted equation will satisfy the regularity conditions of value $z = 1$,

$$W(1)=0,\ DW(1)=0, \Theta(1)=0\ \text{or}\ D\Theta(1)=0\ ,\ \Phi(1)=0.$$

Finally, the critical Darcy Rayleigh number R_{Dc} and the corresponding wave number a_c were obtained numerically in the different forms of $F(z)$ as well as other values of physical parameters of rigid-rigid ferromagnetic (isothermal/insulated) boundary conditions. From (table 3), it was observed that, the numerical results were significantly obtained from the two methods, heterogeneous equilibrium was observed during the model fitting phase.

3. Results

The combined effect of internal heat source and the vertically stratified permeability at the onset of thermomagnetic convection was derived from the Brinkman porous method (heated below temperature). Simulation results have been investigated by driven application of isothermal and insulated rigid ferromagnetic boundary conditions. Present fitted model, we have considered four different forms of vertical heterogeneity permeability function $F(z)$: $F1$, $F2$, $F3$ and $F4$ (table 1). The linear stability problem was solved numerically by using the Galerkin method to know the accuracy of the model, fitted model is very informative to know the provocative values of R_{Dc} and the corresponding a_c at the different levels of Galerkin approximation. The inspection of the results revealed that, the R_{Dc} turn out to be the same in vertically stratified permeability functions of type $F1$ and $F2$ as well as type $F3$ and $F4$, and also the model formulation was vertically stratified with permeability function of type $F4$. Asper the findings F4 was found to be more stable when compared to type $F1$ (ROC analysis was performed to know the accuracy of the model. Asper the model AUC was 0.91 and likelihood function was found to be significantly correlated for propagation of real expected values of permeability function). Further, it was observed that, the values of R_{Dc} will be differ with different vertical heterogeneity of permeability functions at higher order Galerkin method.

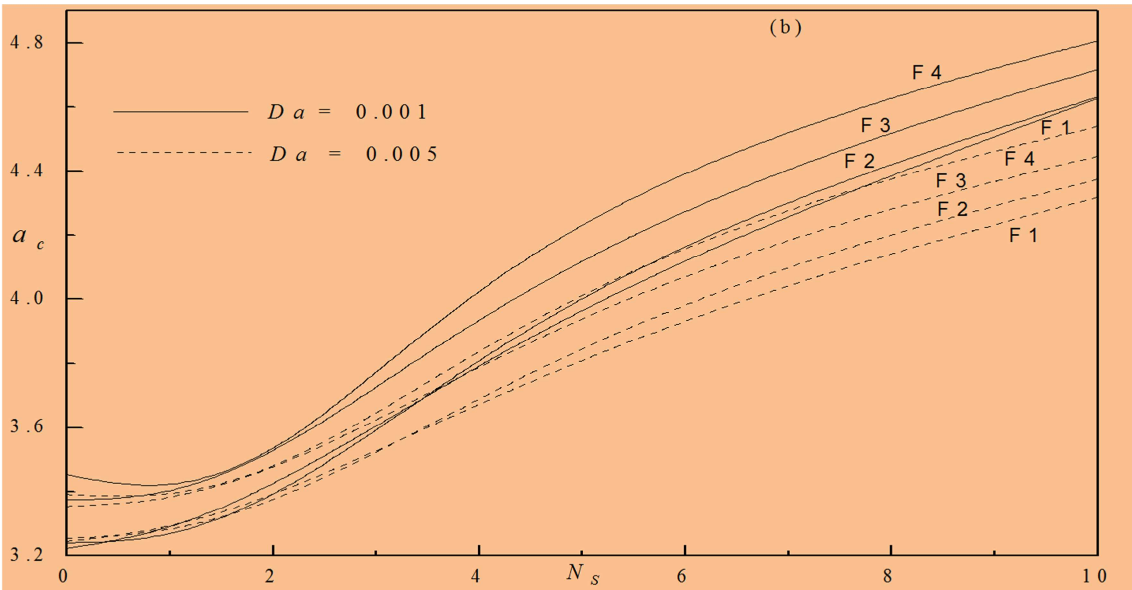

Figure 1. *Variation of (a) R_{Dc} and (b) a_c with N_s for different values of Da when $R_m = 2$ and $M_3 = 1$ for different forms of $F(z)$.*

Table 1. *Various forms of vertical heterogeneity of permeability function $F(z)$.*

Models	α_1	α_2	**Nature of *F* (*z*)**
$F1$	0	0	$F(z)=1$ (homogeneous)
$F2$	1	0	$F(z)=1+\left(z-\frac{1}{2}\right)$ (linear variation in z)
$F3$	0	1	$F(z)=1+\left(z^2-\frac{1}{3}\right)$ (only quadratic variation in z)
$F4$	1	1	$F(z)=1+\left(z-\frac{1}{2}\right)+\left(z^2-\frac{1}{3}\right)$ (general quadratic variation in z)

Table 2. *Comparison of critical Darcy-Rayleigh and the corresponding wave numbers for different orders of approximations in the Galerkin expansion for $R_m = 5$, $M_3 = 1$ and $Da = 0.1$.*

Approximations									
N_s	**Model**	***i=j=1***		***i=j=2***		***i=j=5***		***i=j=6***	
		R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c
0	$F1$	215.708	3.145	216.531	3.147	211.047	3.151	211.047	3.151
	$F2$	215.708	3.145	216.484	3.147	210.983	3.151	210.983	3.151
	$F3$	216.534	3.163	217.308	3.164	211.850	3.168	211.850	3.168
	$F4$	216.534	3.163	217.160	3.165	211.658	3.169	211.658	3.169
2	$F1$	215.815	3.143	208.327	3.198	203.184	3.204	203.164	3.204
	$F2$	215.815	3.143	210.803	3.197	205.529	3.203	205.509	3.203
	$F3$	216.641	3.163	211.532	3.215	206.310	3.220	206.289	3.220
	$F4$	216.641	3.163	213.981	3.214	208.592	3.220	208.573	3.220
5	$F1$	215.976	3.147	177.067	3.384	172.865	3.400	172.800	3.400
	$F2$	215.976	3.147	181.204	3.390	176.798	3.404	176.730	3.405
	$F3$	216.801	3.164	181.623	3.408	177.301	3.421	177.231	3.421
	$F4$	216.801	3.164	185.856	3.415	181.285	3.427	181.212	3.427

Table 3. *Comparison of numerical methods for different forms of* $F(z)$ *and for two values of* N_s *with* $R_m = 5$, $M_3 = 1$ *and* $Da = 1$.

N_s	Model	Galerkin method				Shooting method			
		Isothermal boundaries		Insulated boundaries		Isothermal boundaries		Insulated boundaries	
		R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c
0	$F1$	1748.21	3.120	732.13	0	1748.21	3.120	732.13	0
	$F2$	1748.20	3.120	732.12	0	1748.20	3.120	732.12	0
	$F3$	1749.07	3.122	733.55	0	1749.07	3.122	733.55	0
	$F4$	1749.05	3.122	733.55	0	1749.05	3.122	733.55	0
5	$F1$	1490.13	3.314	728.07	0.555	1490.13	3.314	728.07	0.555
	$F2$	1493.78	3.315	730.87	0.577	1493.78	3.315	730.87	0.577
	$F3$	1494.32	3.317	732.25	0.596	1494.32	3.317	732.25	0.596
	$F4$	1497.97	3.317	735.08	0.616	1497.97	3.317	735.08	0.616
	AUC	0.91**		0.86		0.74		0.72	

4. Model Application

4.1. Medical Science

Medical practitioner highly exhibits tremendous variation in decision making because of their normative approach, biological and clinical essence to deal with uncertainties round the clock. The test diagnostic decision also depends upon the physical experience, expertization and perception of the practitioners. As the complexity of the health care decision system, this model will be insight for taking clinical decision at early stage without any bias for example, blood test and screening of HIV, biopsy test for cervical cancer and identification of rare diseases by using NGS method (Next generation gene sequencing), the F (z) will signify and provides sufficient analytical information to the practitioner at early stage, this system greatly converges large population level and deal with real concept of tremendous approximation of weak law of large numbers. The subtle of the approximation could provide signifying results for the practitioner at inception as well as last stage of decision, and also it can solidify precise what is imprecise in the world medicine. The fitted model shows an important role in medicine for symptomatic diagnostic cures in fuzzification techniques.

4.2. Extension of Artificial Intelligence (AI) in Engineering Science

An artificial neural network (ANN) is an information processing model that is able to capture and represent complex inputs-output relationship. The motivation of F (z) critical Darcy-Rayleigh and the corresponding wave numbers for different orders of approximations in the Galerkin expansion would be extended for AI system that could process information in the same way the human brain. ANNs resemble human brain in two respects learning process and storing experimental knowledge. An AI network learns and classifies problem through repeated adjustments of the connecting weights between the elements.

4.3. Galerkin Expansion (GE) in Bio Informatics

The GE logic can be easily used to implement systems ranging from simple, small or even embedded up to large networked ones. The GE logic is that it accepts the uncertainties that are inherited in the realistic inputs and it deals with these uncertainties in their affect is negligible and thus resulting in a precise outputs. The GE reduces the design steps and simplifies complexity that might arise since the first step is to understand and characterize the system behavior by using knowledge experience. It is successfully applied to several areas in practice like for building knowledge based system of following areas. Increasing flexibility of protein, studying differences between various poly nucleotides, analyzing experimental data sets using GE adoptive resonance theory, aligning sequencing by separate algorithms, complex trait analysis, NGS and Mendelian experimental data analysis etc. [12, 13].

5. Discussion

Asper the resulted findings we have observed that, the linear stability criterion expressed in terms of the critical Rayleigh number (< the number the system is very stable and > the number system tends unstable) [14, 15]. For each of the forms of $F(z)$, the effect of increasing N_s ; R_m and M_3 indicates that, the decreased trend of the Darcy Rayleigh number, while in opposite condition, the trend was significantly increased values of Da . From the (Figure 1.) depicted that, the vertical permeability of heterogeneous function of type $F4$ is more stable followed by type $F3$ and $F2$, least effect was seen in $F1$ with presence of internal heat source ($N_s \neq 0$). The effect of increased internal heat source strength shall express large amount of incremental deviation was propagated simultaneously [16, 17, 18, 19]. In this Juxtapose, the distribution system was induced instability. We have noted that, similar findings were reported by Shivakumara et al and it was found to be significantly associated with permeability heterogeneous function of type $F4$. The least stable was observed in the absence of internal heat source strength ($N_s = 0$) his findings were completely matched with present intervention. The variation of critical Rayleigh number and the corresponding wave number of N_s in different parameter values would be generated heterogeneous simulation figures. The isothermal boundaries of magnetic Darcy-Rayleigh number R_m on the onset of convection will triggered the function of N_s with different forms of $F(z)$ $M_3 = 1$ and $Da = 0.001$, it was found that, the effect of N_s destabilizing the system of homogeneous porous medium considering brief summary of resulted illustration, the increased values of N_s and R_m was found to be positively associated with critical wave numbers. Thus, the wave number effect leads to reduction of convection cells [20, 21]. Further, the inspection of the illustration findings depicted that, the values of a_c was found to be higher in $F4$ followed by $F3$ and $F2$. The insignificant effect was seen in $F1$ in the presence of magnetic Rayleigh number [22, 23].

The non-linearity of magnetization M_3 has not been influence on the onset convection and also the critical wave numbers. Resulted model findings found that, F1 is greatly converges with real values of Rayleigh number and propagates very small values [24, 25].

Variation of critical Rayleigh number of different forms of $F(z)$ was presented in Figure 1 which are demonstrated by two values of $Da = 0.05$ and 0.06 and function of N_s. From the figure 1 we have seen that, the system was more stable in the form of $F(z)$ with type $F4$ followed by $F3$ and $F2$ and least stable equilibrium was observed in $F1$ till observation is turned out to be the same in isothermal boundaries [26, 27]. Besides, an insulated boundary was found to be more destabilizing when compared to isothermal boundaries [28-30].

6. Conclusions

The present study concludes that, the fitted model is found to be more unstable for insulating boundary as compared with isothermal boundary. However, an Increased study state in the value of internal heat source strength N_s, magnetic Rayleigh number R_m and the measure of non-linearity of magnetization M_3 is to be hasten for the onset of ferromagnetic convection, while increasing the Darcy number Da shows stabilizing equilibrium effect on the system due to increased rate of viscous diffusion state.

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